

The Curriculum Design Philosophy and Structure of *Thinkable*: A Free Web-Based Educational Resource for PreCalculus

Abstract

This paper describes the design philosophy and structure of a free online PreCalculus course containing nine units with lessons and assessments. The purpose for developing the resource was to support students and teachers. An explanation is provided of the curriculum design framework that aimed to harness the capabilities of the Internet to create a hands-on, multi-representational mathematics teaching and learning website. The resource was conceived to be pedagogically sound, available and accessible, and to be customizable for the user. The researcher used the perspective of a curriculum designer and of a teacher who sought to provide opportunities for students to interact and engage in their personalized learning trajectories through self-paced learning and assessments. The resource was also designed to be adaptable by teachers for their own practice. Future research will assess how students and teachers use the resource and which design and learning theories were supported by the results. (149 words)

Introduction

The goal in this design experiment was to develop lessons that used the Internet's animation and interactive capabilities to address mathematical pedagogies for high school content and learners. Although a number of secondary mathematics resources can be found online, at this time, there is no resource that is free and links curriculum learning outcomes to lessons that are fully customizable, interactive, and contain complete planned lessons with assessments. A call has been made for flexible and customizable learning platforms (e.g., Kopp & Crichton, 2007). Websites, although ever changing, tend to address mathematical content using a lecture-based approach, and generally apply algebraic methods and procedures using examples (e.g., Khan, 2011). Alternatively, some sites contain activities or interactive objects that are scattered across domains or are costly to join (e.g., Benesse America Inc., 2012; ExploreLearning, 2012). With regards to secondary mathematics web content that is available today, the general focus is either on the sharing of materials or on producing high quality resources; however, rarely, is the combination of high quality pedagogy integrated with the optimal uses of the Internet's capability. Quality resources for teaching and learning secondary mathematics are especially lacking (Van de Walle, Folk, Karp, & Bay-William, 2010). Access, together with deep and interactive "meaning-making" still need to be addressed (Hall, Watkins, & Eller, 2012; Hoadley & Van Haneghan, 2012). The goal of this online resource will be to provide free web-based support for students and for teachers where

high-quality learning merges with optimal use of technology. It is expected that *Thinkable* will be launched online in early to mid-2013.

The Context of the Design Experiment

Thinkable is designed to be a web-based instructional platform offering mathematics course support free of charge for students and teachers, starting with PreCalculus Mathematics 11. It addresses some important constructivist practices such as students' active engagement with learning objects and students' distinctive learning styles (Gardner, 1993; Hall, et al., 2012; Sternberg, 1997) and includes some direct instruction with scaffolding (Hoadley & Van Haneghan, 2012). The design considers multiple perspectives based on various learning goals in order to determine which features teachers and students prefer (Clark & Hannafin, 2012). Learners can experience a deeper understanding from interacting with multiple representations to make sense of mathematical concepts (Fosnot, 1996).

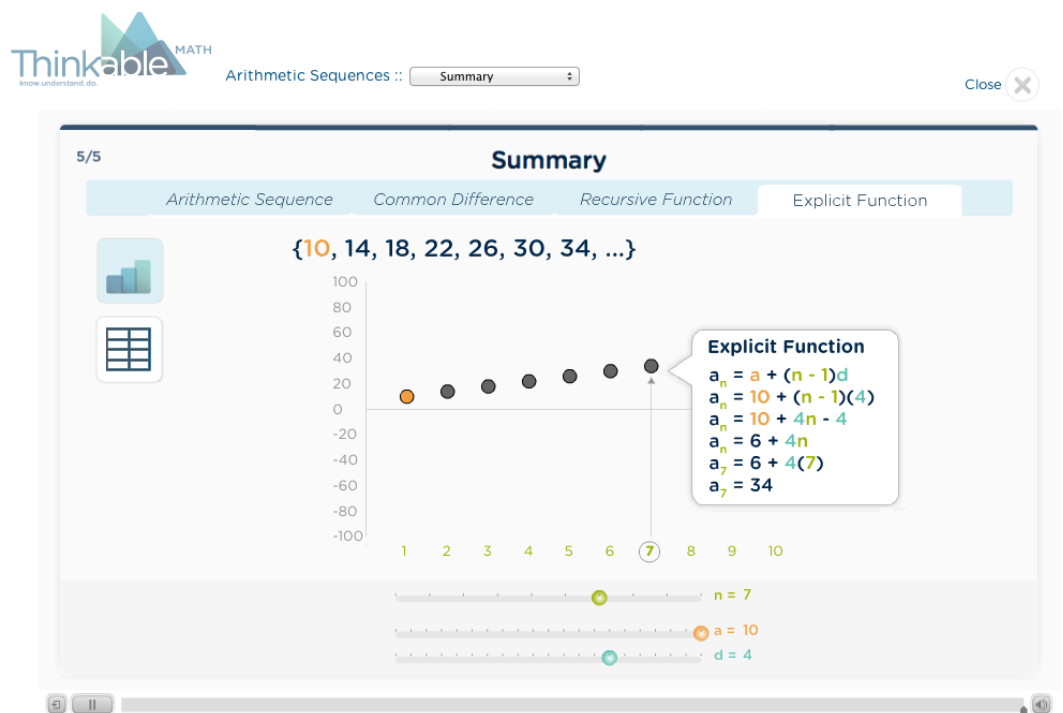


Figure 1. Screen shot of prototype: Summary page with multiple representations

Thinkable is designed to maintain a focus on understanding concepts rather than on simple computation or procedural mathematics skills. This is accomplished through the invitation to play with images that are interactive and represent ideas in multiple ways, including graphically, numerically, and algebraically. The

power of the interactive object or *applet* is that the user can proceed through many examples by changing one parameter at a time, and, simultaneously, see the impact on the patterns that emerge in the various representations on the screen. The user is in control of the interactive image and can learn to see the patterns at her own pace. As Hall, et al. (2012) suggested, students can “learn complex information most effectively if they are allowed to experience the information in various formats” (p. 371). One of the goals of this design is to dispute the image of mathematics as simply computation, processes, algorithms, rules, and drills, rather than a “way of thinking, and a set of intricately connected higher-level skills” (Paulos, 2001, p. xiii). Vanderbeeken (2009) called for learning to be “hands-on, experience-based, multi-disciplinary, physical, and enabled by immersive technologies” (p. 57). He described how some good examples were available online, but that the “educational, pedagogical approach is lacking” (p. 57). *Thinkable*’s approach is novel in its attempt to attend to constructivist and other learning theories and practices through a web interface, particularly with the application of meaningful and multi-modal concept development in higher mathematics.

Design Research Methodology

According to Schoenfeld (2009), the goal of design research is to develop “materials that make a difference” (p. 4). Design research is essentially interventionist (Kelly, 2006), and in this design experiment, the educational intervention is the interaction of students and teachers with the *Thinkable* website. The overarching question guiding this research is whether or how this approach might make a difference in learning and teaching mathematics. It simultaneously re-addresses what it means to do, understand, or learn mathematics.

This research will adopt a design research approach. The process of design research is rigorous and complex as is the role of the design researcher (e.g., Burkhardt, 2009; Hoadley & Van Haneghan, 2012; McKenney, Nieveen, & van den Akker, 2006; Reeves, 2006; Schoenfeld, 2009; Walker, 2007). Walker (2007) described design research with technology as follows:

In design research, a theorist or researcher’s rigorous analysis of a learning problem leads to quite specific ideas for interventions. Designers then build systems that use information technology to create specific teaching and learning materials and methods designed to realize learning gains predicted

by theory and research. If the theoretical analysis is right, then these interventions ought to give markedly more effective results. (p. 9)

Design research must begin with a clear theoretical position and a practical goal. Then, as Walker reminded us, the intervention must be analyzed rigorously, similar to engineering research, to test the intervention in real applications:

Designing these systems is an R&D endeavor, not a work of imagination or a straightforward deduction from theory. In order to create interventions, designers need to study how students and teachers actually responded to specific features of the design suggested by the theory. In other words, to show that design rigorously implements principles from research and theory, designers must do design research. (p. 9)

Walker (2007) prescribed the necessary framework for the research questions and methodology of a design experiment such as this one. Prior to framing this design experiment's theoretical and methodological approach, a description of the intervention is necessary.

The Design of the Resource

The structure of each lesson is described here. There is a well-structured hierarchy (Hall, et al., 2012) comprised of nine units in the Pre-Calculus 11 course, and each unit consists of four to nine lessons. Each lesson is framed with a *hook*, or anticipatory set, and five slides. Hall, et al. described the importance of lessons being broken down into smaller parts, and the benefits of “relevant dynamic graphics” (p. 371) interspersed with multiple presentations. All of these features are included in the design of this resource.

Hook

Each unit begins with a hook. This is an entertaining animated video that introduces the concepts in the unit using an authentic situation that intends to justify the need for the mathematics to be learned. It is important for designers to work towards students' perceiving the initial activity as *real* (Hoadley & Van Haneghan, 2012). This can be a difficult undertaking, as Figueiredo, van Galen, and Gravemeijer (2009) noted: “What is real for a particular student, however, does not have to be real for another student or for the teacher” (p. 3).

Lesson: Five slides

- i. Each slide follows principles of web design and presentation appeal, which implies that elements are revealed in groups and sequentially, the background contains sufficient white space, all objects are large enough to be readable (Williams, 2000), and text is limited (Hall, et al., 2012). The slide has uniform density, and not too much information is captured on each screen (Parush, Shwartz, Shtub, & Chandra, 2005).
- ii. Each slide includes a simultaneous explanatory narration. Narration has been shown to bring cohesiveness and meaning to understanding (Hall, et al., 2012; Weller, 2000) Moreover, “[e]vidence suggests that well-designed graphics and animations improve learning and that narration enhances learning from graphics” (Cook, 2007, p. 40). Hall, et al. (2012) indicated that “the learner should be provided with guidance (p. 370).
- iii. Each slide ends with an applet or an animation. It is important that users can physically and actively engage with the learning object (Shang, Shi, & Chen, 2001). Hamada (2007) noted that “visual tools can enhance learners’ motivation and performance” (p. 64). Hall, Watkins, and Eller (2012) reminded us that the use of multimedia simulation and interactivity is effective.
- iv. Each slide ends with a Prescribed Interaction (PI) that guides the learner towards understanding the connections made through the applet. This has the dual purpose of providing formative feedback to the learner; the user is prompted to re-try or continue. Anderson (2003) explained that “attention to ways to create ‘guided didactic interaction’ in the text materials can create high levels of student-content interaction” (p. 7). Hall, et al. (2012) reminded us that “learners learn most effectively when they are actively engaged in learning, as opposed to passively reading or listening” (p. 371).
- v. Each slide concludes with a Focus Question (FQ) that formatively assesses a basic conceptual understanding of the slide before the user moves on to the next concept in the lesson. Opportunities for formative assessment have been shown to be important design features for web learning (Hall, et al., 2012). Feedback is given during this formative assessment when students select correct or incorrect answers (Black & Wiliam, 1998; Clark & Hannafin, 2012; Stiggins, 2001, 2007).

Practice and assessment

The practice section, like the lesson slide design, deliberately addresses conceptual understanding in a progressively more complex manner (Hall, et al., 2012). The first goal is deep understanding of conceptual mathematics; however, because the resource also addresses preparation for future assessments and

students' ability to solve problems, these goals have been reflected in the four choices for assessment that have been developed.

- i. ***Formative assessment during the lesson*** is achieved through the PI and the FQ as described in part iv) and v) above. These check students' basic conceptual understanding and invite them to engage while they navigate through the lesson.
- ii. ***Formative assessment through practice*** is addressed using six or seven computer-scorable questions as described below. This feature will allow students to self-assess, prepare for future tests, and software features will permit teachers to check to what extent students have completed this practice.
- iii. ***A challenge question*** is available after the practice has been completed if students wish to engage in it. It is intended to be optional, and is available for students and teachers who desire a more difficult or advanced question.
- iv. ***Rich contextual tasks*** are available if a teacher or student prefers to teach or learn through problem-solving. The class tasks are designed so that teachers can introduce a rich contextual problem that the students in the class can solve with more limited guidance from the teacher. The tasks are open, and some information might be superfluous. There are many ways to complete the task, and the task addresses many learning outcomes at once. A rubric is provided that helps students self and peer assess their work. Generally, these tasks are solved collaboratively, but they could also be solved individually.

This balance of assessment choices reflects the position that “[t]oday’s and tomorrow’s designers must support different kinds of learning” (Clark & Hannafin, 2012, p. 376), and that we must recognize alternative perspectives, methods, beliefs and approaches to design (Clark & Hannafin, 2012). This balance of learning and assessment methods also supports Hoadley and Van Haneghan’s (2012) assertion that the learning sciences must be “open to multiple perspectives on learning” (p. 56).

The computer-scorable *Practice* section of the site is based on the most advanced work available for testing knowledge and understanding using technology. A protocol for designing the practice questions is described in Appendix A. These types of machine-scorable questions address students' basic skills and understanding by applying the Cognitive Processes, which are based on a revision of Bloom's Taxonomy by L. W. Anderson and Krathwohl (2001). The creation of these types of questions is supported by the University of Iowa, with the goal of developing machine-scorable questions that address lower- and higher-

order thinking skills. Samples of these questions, designed by Scalise and Gifford (2006) are available on Scalise's (2012) website. These question-types were designed and funded by partnerships such as the Smarter Balanced Assessment Consortium (SBAC) (2010) and the Partnership for Assessment of Readiness for College and Careers (PARCC) (2010). As progressive as they are, while striving to measure higher-level understanding, they were essentially designed using an instrumental and assessment-driven perspective which includes goals such as the development of a common assessment system that "will help make accountability policies better drivers of improvement" (p. 8). Though this research does not take a perspective of accountability, the ability of students to engage in *technologically dependent* assessments will likely remain a reality in their futures, and this resource is designed to provide access for students to succeed within the current and predicted assessment culture in education. Furthermore, future studies involving *Thinkable* will assess to what extent users, including both students and teachers, preferred each type of assessment.

A challenge question is offered for users who prefer an accelerated approach to learning. The *Brain Cracker* is available once users have completed the *Practice* section. This question is designed to be optional, and is more complex than what would normally be expected for the given learning outcomes of that lesson. Again, these options are intended to offer choice for the users.

For teachers or students who wish to take a problem-solving or more constructivist approach to learning, rich contextual tasks have also been developed (Burkhardt, 2012). Burkhardt also noted that well-designed assessment includes "short items and substantial performance tasks so that teachers who teach to the test, as most teachers will, are led to deliver a balanced curriculum that reflects the standards" (p. 2). Samples of rich tasks are available in online repositories (e.g., Centre for Mathematical Sciences, 2012; Meyer, 2012; Shell Centre for Mathematical Education, 2012). A number of class tasks will be designed for each unit of the *Thinkable* resource. The class tasks are accompanied by a generic rubric that can be used to assess the tasks, either by learners, their peers, or their teachers. This feature helps to complete the variety of types of activities and assessments that learners or educators may choose.

Theoretical and Conceptual Framework

The following theoretical intent of the design experiment describes why this particular learning process was chosen. Pirie and Kieren's (1994) model of understanding is invoked as an on-going, recursive, and iterative process where learners develop *thicker understandings*. This respects a truly *learner-generated knowing* intended by the ideals of constructivism. The perspective of Realistic Mathematics Education (RME) is also reflected in this design experiment as it seeks to create "opportunities for students to come to regard the knowledge they acquire as their own knowledge" (Boon, 2009, p. 1). The technological aspect of the resource also adds the future possibility of studying the collective action and social aspect of learning through *Thinkable*; however, this aspect of the study has yet to be fully described or defined. Prior to investigating the social networking possibilities of the site, the primary research study must examine how students and teachers use the site.

The learning theory perspective for *Thinkable* was focused on learner understanding and meaning-making (Hoadley & Van Haneghan, 2012). These ideas were first introduced by Dewey (1933), and, particularly in mathematics education, by Fosnot (1996) who promoted understanding, and also by Paulos (2001) who argued that any learner can understand mathematics. An emphasis on understanding includes building on prior knowledge (Black & Wiliam, 1998; Swan, 2008), the importance of making connections and of confronting the complexity of mathematics (Swan, 2008), and attention to meaning-making in mathematics (Hiebert et al., 1997). Other aspects of a constructivist learning theory in mathematics education are supported by Vygotsky's (1964) emphasis on the importance of language for learning (also supported by Hoadley & Van Haneghan, 2012), von Glasersfeld's (1995) highlighting students' responsibility for learning, and Prawat and Floden's (1994) contribution regarding the need to develop students' confidence in mathematics.

It is also important to consider the choice of using interactive applets in the design. Again, Dewey's (2002) emphasis on active learning grounds others' work, including von Glasersfeld (1995), Fosnot (1996), and Swan (2008) as previously mentioned. Eisner also (1998) noted that: "[t]he act of representation is also an act of invention. Representation is not a monologue; it is a dialogue between the individual actor and the material acted upon" (p. 18), and furthermore that: "[t]he

ability to ‘make sense’ out of forms of representation is not merely a way of securing meaning – as important as that may be – it is also a way of developing cognitive skills” (p. 8).

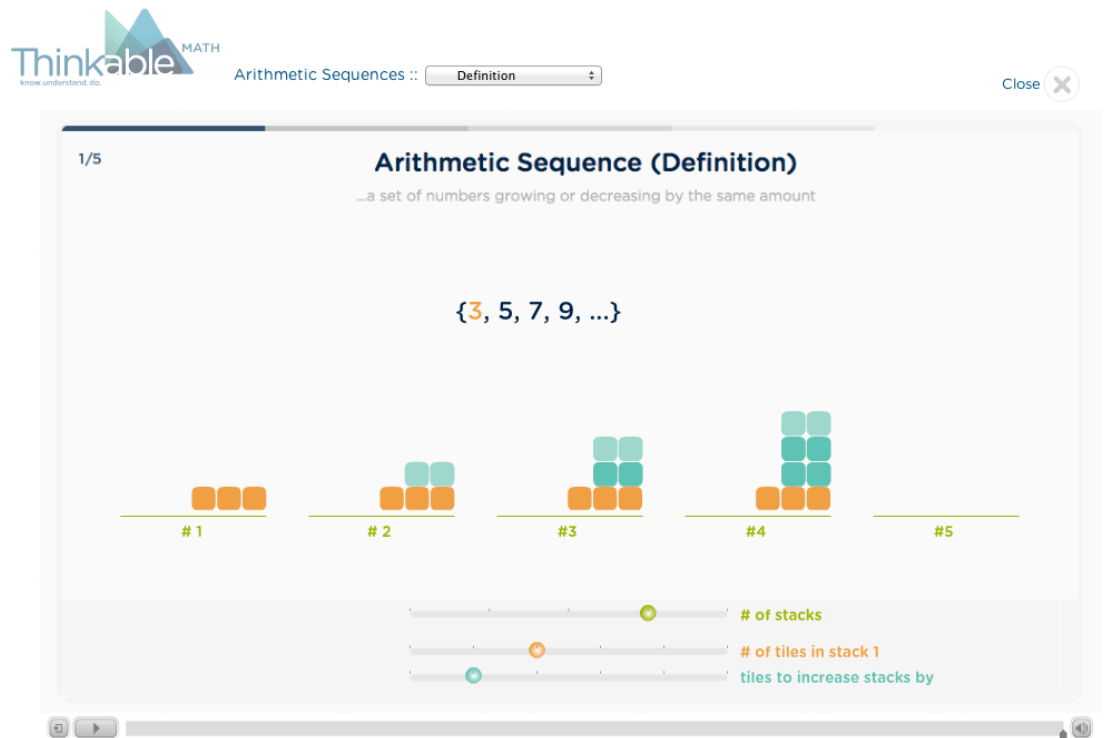


Figure 2. Screen shot of prototype: First of five slides with interactive applet

The lesson developers were practicing teachers who worked collaboratively with this researcher to design the raw content of the lessons. Hoadley and Van Haneghan (2012) and Sawyer (2006) indicated that those who design instruction need to:

- focus on the development of learners’ conceptual understanding,
- put learning processes on par with teaching processes,
- aim for authenticity of instruction,
- build on learner’s prior understand, and
- provide opportunities for learners to engage in reflection.

As design researchers, guided by our theoretical perspectives, we were *bricoleurs*, and tinkers, who invented new applications and adapted existing materials. A flexible social constructivist lens was espoused (Berger & Luckmann, 1966) during the development of this resource. There is a study in progress that investigates the design process for this resource.

All in One Place and Complete

One advantage of the *Thinkable* resource is that it presents a comprehensive body of resources for the high school curriculum in a single intuitive interface. It is designed to be methodical, trustworthy, consistent, and easily navigable. Lessons can be opened directly through an Internet search, or through a clearly structured navigation system that can be accessed by location (Province or State) and by course. If users choose to log in, their learning is tracked for them, and they can continue from where they left off.

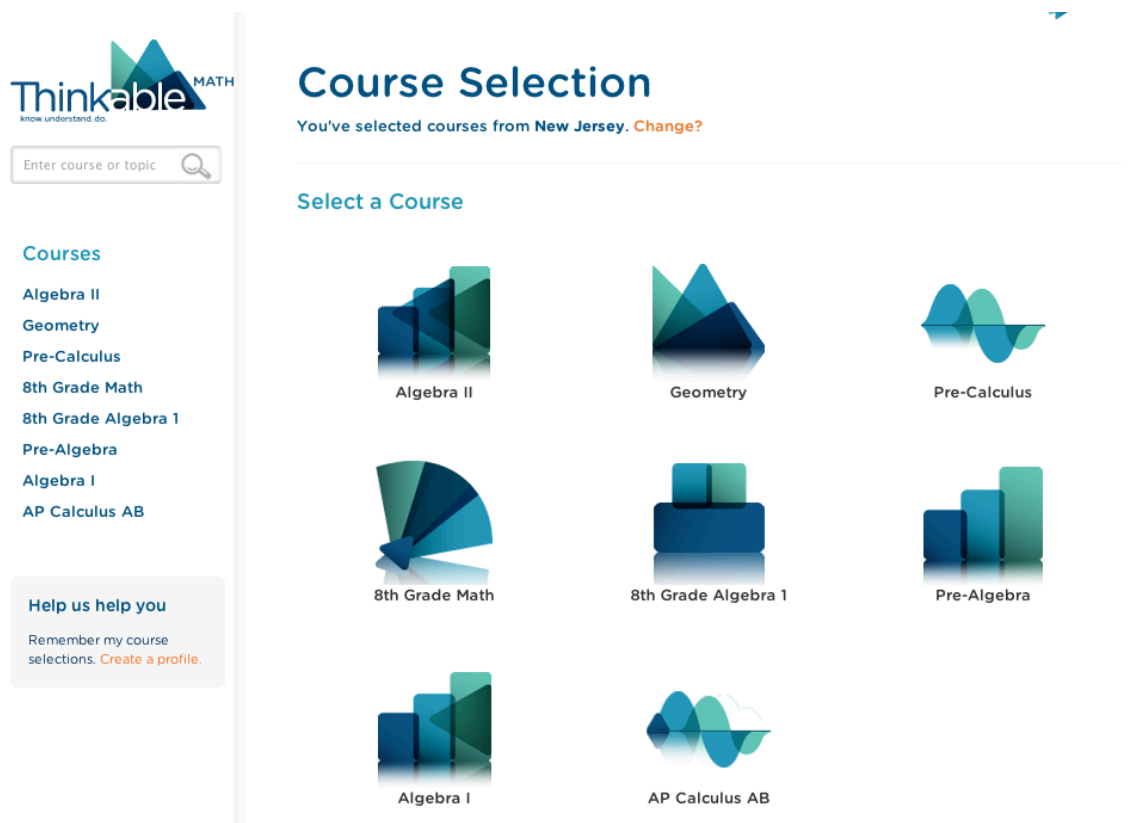


Figure 3. Screen shot of prototype: All courses easily accessible and navigable

Linked to Courses and Curriculum

The question might be asked: “What mathematics matters?” (Schubert, 2008). The *Thinkable* design began with the pragmatic view that the Common Core State Standards and the WNCPC curricula contain necessary learning outcomes for students in today’s world. These learning outcomes became the starting points for lesson development. The pre-determined U.S. or Canadian learning outcomes are an appropriated place to begin. From an access perspective, curriculum is becoming more globalized and uniform around the world

(Anderson-Levitt, 2008), and it can be argued that the North American curriculum documents reflect this common direction.

Free and Accessible

Innumeracy, or “an inability to deal comfortably with the fundamental notions of number and chance, plagues far too many otherwise knowledgeable citizens” (Paulos, 2001, pp. 3-4). It is becoming evident that quantitative literacy is necessary for fully contributing citizens of tomorrow. The definition of Quantitative Literacy (QL) used in this design experiment offers an emancipatory perspective for teachers and for their students:

Quantitatively literate citizens need to know more than formulas and equations. They need to understand the meaning of numbers, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. QL empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are the skills required to thrive in the modern world. (Steen, 2001, p. 2)

Furthermore, there is a pervasive misconception that mathematics is only for the few. Paulos (2001) described the situation as follows:

Of course, some people have more talent than others in mathematics, just as some write better than others, but we don't advise students to forget their English and literature courses if they're not planning to be journalists or novelists. Almost everybody can develop a workable understanding of numbers and probabilities, of relationships and arguments, of graphs and rates of change and of the ubiquitous role these notions play in everyday life. (p. xiv)

If quantitative literacy is both necessary and possible in today's world, then every learner can and should have access to the ideas and concepts of mathematics at a sufficient level of complexity so that they can more fully engage as confident and productive citizens. *Thinkable* is designed to provide this access for all learners.

Adaptable

The design of *Thinkable* allows users to access the materials in any order and for many purposes. Students may choose to listen to the narrated lesson. Teachers may choose this route as well; they can preview the lesson and use any

sections they feel are of value. Other users will choose to learn by directly playing with the applets, or by accessing the self-paced Practice questions.

Future Research Questions

In future studies, the following questions will be considered:

- 1) How do students use *Thinkable* for learning?
 - a) Which design features are most appreciated and why?
 - i) attention to Learning Styles and Multiple Representations
 - ii) choice and adaptability
 - iii) ability to interact with applets and learning objects
 - iv) practice
- 2) How do teachers use *Thinkable* for teaching?
 - a) Which design features are most appreciated and why? (same sub-questions as above)
 - b) Do teachers use *Thinkable* for their personal professional learning? If so, how?
 - c) Do novice teachers, or “out-of-field” teachers use the site differently than experienced or expert teachers? If yes, then how?
- 3) Which design or learning theories do the results of the study support?

Other studies are already in progress with regards to this product. The process of lesson design and development will be more closely examined. As Kopp and Crichton (2007) suggested, it is important to investigate “learning and the actual practices of educators as they attempt to use [learning objects] in the field” (p. 3) . When studying classroom use in particular, Hoadley and Van Haneghan (2012) and Bransford and Schwartz (1999) recommend that classrooms be designed to be “learner centered, knowledge centered, feedback/assessment centered, and community centered” (p. 59).

Summary

The purpose of this paper was to describe the curriculum design framework for *Thinkable*. The resource is a free online PreCalculus course containing nine units with lessons and assessments. The purpose for developing *Thinkable* was to support students and teachers. This paper described the curriculum design framework that aimed to harness the capabilities of the Internet to create a hands-

on, multi-representational mathematics teaching and learning website. The resource was conceived to be pedagogically sound, available and accessible, and to be customizable for the user. The researcher used the perspective of a curriculum designer and of a teacher who sought to provide opportunities for students to interact and engage in their personalized learning trajectories through self-paced learning and assessments. The resource was also designed to be adaptable by teachers for their own practice. Future research will assess how students and teachers use the resource and which design and learning theories were supported by the results.

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Appendix A: Protocol for the Design of the Practice Questions

Appendix A: Protocol for the Design of the Practice Questions:

For each Lesson:

SIX (6) Machine-Scorable Questions:

~ONE (1): Basic Knowledge (Remember)

~THREE (3): Understanding

~TWO (2): Do (Apply, Analyze or Evaluate)

Some questions should be contextual and preferably related to the hook.

*The highest Bloom's level, *Create*, will be assessed using Class Tasks and Unit Tasks, and through future iterations of *Thinkable* using social networking capabilities.

Use key words from Cognitive Processes dimensions chart below to decide which level is being addressed. Retrieved from: <http://www.celt.iastate.edu/pdfs-docs/teaching/RevisedBloomsHandout.pdf>

Table 2. The Cognitive Processes dimension — categories & cognitive processes and alternative names

lower order thinking skills			higher order thinking skills		
remember	understand	apply	analyze	evaluate	create
recognizing <ul style="list-style-type: none"> identifying recalling <ul style="list-style-type: none"> retrieving 	interpreting <ul style="list-style-type: none"> clarifying paraphrasing representing translating exemplifying <ul style="list-style-type: none"> illustrating instantiating classifying <ul style="list-style-type: none"> categorizing subsuming summarizing <ul style="list-style-type: none"> abstracting generalizing inferring <ul style="list-style-type: none"> concluding extrapolating interpolating predicting comparing <ul style="list-style-type: none"> contrasting mapping matching explaining <ul style="list-style-type: none"> constructing models 	executing <ul style="list-style-type: none"> carrying out implementing <ul style="list-style-type: none"> using 	differentiating <ul style="list-style-type: none"> discriminating distinguishing focusing selecting organizing <ul style="list-style-type: none"> finding coherence integrating outlining parsing structuring attributing <ul style="list-style-type: none"> deconstructing 	checking <ul style="list-style-type: none"> coordinating detecting monitoring testing critiquing <ul style="list-style-type: none"> judging 	generating <ul style="list-style-type: none"> hypothesizing planning <ul style="list-style-type: none"> designing producing <ul style="list-style-type: none"> constructing

(Table 2 adapted from Anderson and Krathwohl, 2001, pp. 67–68.)

Cover all learning outcomes. Indicate the main learning outcome addressed by each question.

See “Practice-Mate” for working chart to create specifications table.

Practice question writers would become familiar with the types of questions as listed at:

<http://pages.uoregon.edu/kscalise/taxonomy/taxonomy.html>

SUGGESTED To Assess:	Use:
Knowledge	1C, 1D, 3A, 4A, 4B, 2A, 2B, 2C
Understanding (e.g., Interpret)	2A, 2B, 2C, 1A, 1B, 2D, 3B, 3C
Do: Apply	5A, 6A
Do: Analyze	4C, 5D
Do: Evaluate	3D, 5C